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ABSTRACT

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THE USEFULNESS OF SELECTION TESTS

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Introduction

In many admissions decisions a selection test is used to decide which members of an applicant pool will be selected for the available openings. The correlation coefficient between the selection test and the college's criterion of success, Grade Point Average, say, are often in the range of .40 to .55 for the applicants who are selected and then enroll. Such correlations are often squared and then interpreted as the proportion of variance explained by the predictor. Such a statement is true but is very difficult for an admissions officer to relate to what the effect is on his entering class. Some better interpretive scheme is needed.

The purpose of this paper is to show the effect of using a selection test on the average criterion score of the entering class. The correlation coefficient (times 100) is shown to be the percentage of improvement of using the selection test over what would happen on the average if the test were not used. Further, a simple formula is developed for approximating the probability that selection by chance would yield an entering class as able or abler than a class selected with the help of a valid predictor. It is further shown that, for reasonably sized colleges and with reasonable assumptions, selection using even small correlations will almost certainly result in an entering class that would earn higher criterion scores than would occur without the selection test.

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1. The Effect of Different Selection Procedures

Let us assume that an admissions officer has the task of filling n available openings in a college from a pool of N applicants. We will call the selection ratio $f = n/N$. We will assume throughout that his aim is to select the "best" applicants in the sense that they are the students who will perform best on some criterion of college success such as grade point average (GPA). The score on the criterion variable is not, of course, known at the time of admission decision and is never known for applicants who are not accepted and/or do not enroll.

Let us now number the applicants with an index i $(i=1,2,\ldots,N)$ and name the criterion variable y. The applicant pool is the population of interest; we will not assume that it is a sample from some larger population. The score of applicant i on the criterion variable y is y_i which is the score that the applicant would receive on the success criterion if the applicant were selected and did enroll. For convenience and without loss of generality, we will consider the values y_i to be in standard form, that is, to have an average value $\bar{y}$ of zero and a standard deviation S of one. We do not assume, at this point, that the distribution of the y_i is normal.

Let us first consider what would happen in some extreme selection situations. First, let us assume that the admissions officer is prescient, that is, that he knows exactly how each applicant would perform on y if admitted and enrolled. In this case, the admissions process would be straightforward; the n candidates who would receive the highest y_i would

be accepted and in this way the average score on the criterion would be maximized. The decision rule is clear, except for tied y_i at the cut-off point which could be selected randomly. We will call the group selected this way the optimum group and the mean criterion score of that group will be called $\bar{y}_1$. Another extreme admissions situation is the case in which the admissions officer knows nothing about the applicant pool, at least nothing that is related to the success criterion. We can consider this case as knowing the applicants index number, i, and no more. In this case, the admissions officer might select applicants at random and hope for the best. There is a tiny probability that the resultant admittees would be the optimum group, but there is also a tiny probability of selecting the group with the lowest possible average score on y. Yet, with no useful predictive information available, there would be little else that could be done.

However, although we do not know what <u>will</u> happen if the applicants are selected at random, we do know everything that <u>could</u> happen. First, we know that there are precisely

$$C = \frac{N!}{n!(N - n)!} \tag{1}$$

different ways in which n students can be selected from a pool of N applicants. Let us number them with the index c $(c=1,2,\ldots,C)$. Each possible selection would result in a mean score for the criterion which we

will label $\bar{y}_c$. We also know that the average of all possible $\bar{y}_c$ is

$$\text{ave}(\bar{y}_c) = \bar{\bar{y}} = 0 \tag{2}$$

that is, not surprisingly, that the average of all samples of size n is the average of the applicant pool which is zero. We also know that the variance of the C possible $\bar{y}_c$ is

$$\text{var}(\bar{y}_c) = s^2\left(\frac{1}{n} - \frac{1}{N}\right) = s^2\left(\frac{1 - f}{n}\right) = \frac{1 - f}{n} \tag{3}$$

since the variance of the y_i is unity. Proofs of the average and variance are shown in Cochran [1977]. If the student selection were at random, then any of the C groups is as likely to occur as any other, and thus the average mean criterion score over a large number of random assignments would approach zero. We will label the average mean as $\bar{y}_0$.

Let us now consider a less extreme--and more realistic--situation in which the admissions officer has some imperfect information about the applicants. Let us assume, for example, that each applicant's dossier contains his score on an admissions test. We will call that test x and the score of applicant i on that test x_i ; x_i is known for all applicants. For convenience, we will also assume that the scores on x are in standard form with zero mean and unit variance. Let us also assume that, from experience,

we know that the linear correlation of x with y is ρ. We will assume throughout that $\rho \geq 0$. One possible admissions rule is to select the n students with the highest scores on x. The question we wish to explore here is how much better will the admittees be if selected using x rather than selecting at random.

We do know something about what will happen if x is used for selection. First, we can partition the criterion scores into two parts

$$y_i = \rho x_i + e_i \tag{4}$$

where the first part ρx_i is the "predicted" value of y_i and e_i is a residual. Let us call the average y_i for the selected group $\bar{y}_\rho$. If we select the n applicants with the highest scores on x, then the value of

$$\bar{y}_\rho = \frac{\rho}{n} \Sigma_+ x_i + \frac{1}{n} \Sigma_+ e_i \tag{5}$$

where Σ_+ means summing the scores of the applicants who have the top n values of x. If the relationship between y and x is linear, that is, the conditional mean of y given x is ρx, then the term $\frac{1}{n}\Sigma_+ e_i$ will vanish and thus we may say

$$\bar{y}_\rho = \rho \bar{x}_+ \tag{6}$$

where $\bar{x}_+$ is the average of the top n applicants.

Comparison of the average criterion score of selected groups under optimum information, $\bar{y}_1$, no information, $\bar{y}_0$, and under information from some correlate, $\bar{y}_\rho$, leads to an interesting interpretation of the correlation coefficient which was first proposed by Brogden [1946]. Under the assumption that the distribution of y is the same as the distribution of x , then the average of the top n scores on y is the same as the top n scores on x , although the actual persons having those scores may be different. Under this assumption, therefore, $\bar{x}_+ = \bar{y}_1$, and

$$\rho = \frac{\bar{y}_\rho}{\bar{y}_1} \tag{7}$$

that is, the correlation coefficient (not squared) is the ratio of the mean of the group selected using x to the mean of the optimum group. Since the average mean of random selections is zero, 100ρ may be interpreted as the percent of possible improvement in selection attained by using the selection test over what would happen, on the average, using random or any arbitrary selection procedure that is independent of y .

Figure 1 shows the relationships among $\bar{y}_0$, $\bar{y}_\rho$, and $\bar{y}_1$ in the case where the top 10% of the applicants are to be selected. The abscissa represents scores on y and the ordinate is proportional to the number of applicants with a particular score. The normal distribution was selected for pictorial purposes. The shaded portion of the curve represents the top 10% of the applicants which has a cut-off score at +1.28. The average score of the top 10% is a +1.755 which is the mean of the optimum group, $\bar{y}_1$. The average random selection, $\bar{y}_0$, is zero. The line on the graph between $\bar{y}_0$ and $\bar{y}_1$ has tick marks to show where the mean of the selected

group would be if the correlation with the selection test, ρ , were +.10,.20,...,.90. If $\rho = 0$, then the effect of the selection test would be the same as the $\bar{y}_0$ and if $\rho = 1$, the effect is the same as $\bar{y}_1$.

Insert Figure 1 about here

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2. The Probability of Exceeding $\bar{y}_\rho$ by Chance

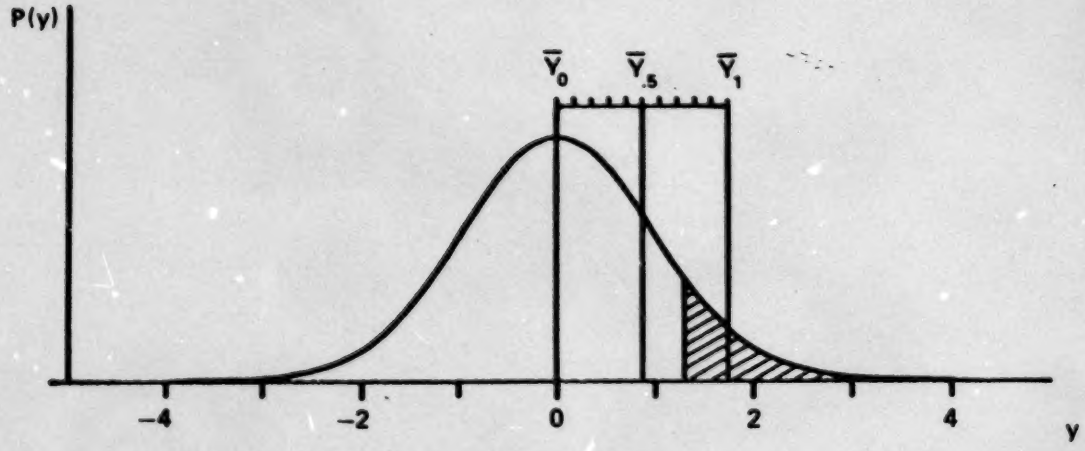
The improvement of the mean of the entering class using a small correlation, $\rho = .10$ or .20, say, may not, at first, seem worth the cost of the testing effort, but this is seldom so. The alternative would be to leave the selection to chance, since there is at least some probability that the chance selection would be better than the selection by a test. The question to be asked now is: what is the probability of actually selecting a sample by chance which results in an average y as high or higher than the mean of those selected by a test? The answer to this question is conceptually simple since all we would have to do is enumerate all C possible samples, count the number of samples which have means as high or higher than $\bar{y}_\rho$, and divide that number by C to find the proportion as high or higher than $\bar{y}_\rho$. However, this direct solution is not feasible since even for a small problem like selecting 10 or 90 out of a sample of 100 has a value of C is approximately 1.73×10^{13} . However, a reasonable approximation to that proportion is possible and used here.

The ability to compare random selection against selection by x gives an opportunity to show the importance of even small correlations in selecting

FIGURE 1

DISTRIBUTION OF APPLICANT POOL

$\bar{Y}_0$ IS THE AVERAGE OF ALL RANDOM SELECTIONS
$\bar{Y}_{.5}$ IS THE AVERAGE OF GROUP SELECTED WITH $\rho = .5$
$\bar{Y}_1$ IS THE AVERAGE OF THE OPTIMUM GROUP
$f = n/N = .1$
CROSSHATCH AREA REPRESENTS TOP 10% OF DISTRIBUTION



an entering class. Figure 2 shows the probability of doing as well or better by chance than by a selection test using the assumption that the distribution of y in the unselected population was normally distributed with zero mean and unit variance.

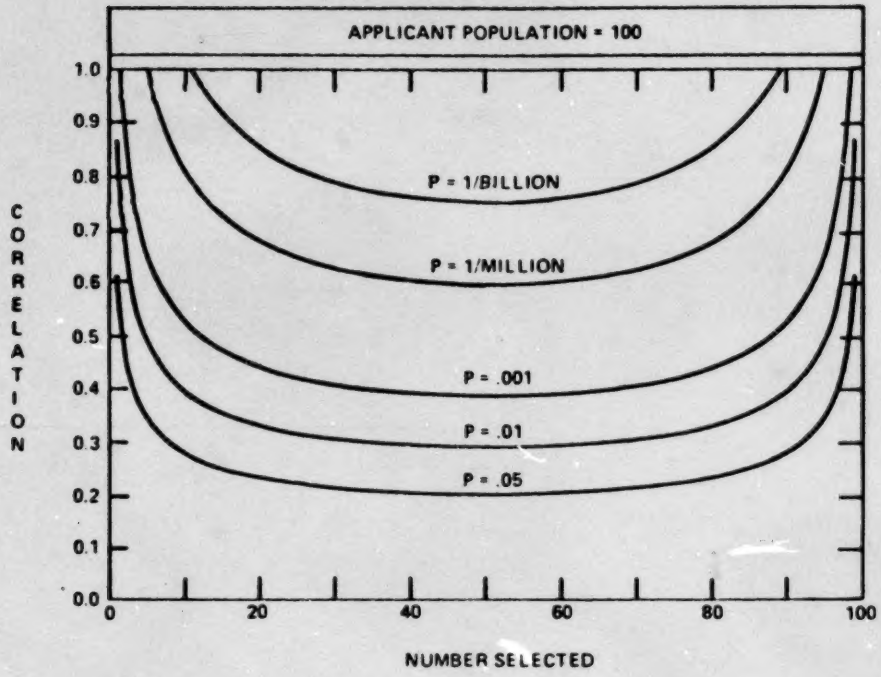
Probabilities are graphed for applicant pools of 100 and 1000, for different selection ratios, and for different values of ρ . We see that, in selecting 50 out of 100 applicants, a correlation of .30 is sufficiently high that chance selection would yield a higher mean than selection using the test not more than one time in a hundred. If the correlation is .40, the selected group may range from 10 to 90 and the probability that a group selected by chance will not excel the predicted group will not exceed one in a hundred. With an applicant pool of 1000, a correlation of .10 is large enough so that the probability of selecting a group of 500 with a larger mean by chance selection is less than .01 and a correlation of .20 is large enough so that the probability of selecting a group with a larger mean by chance is about 1/1000 or less. Thus, the use of a test with even a modest correlation will almost certainly result in a higher average of y .

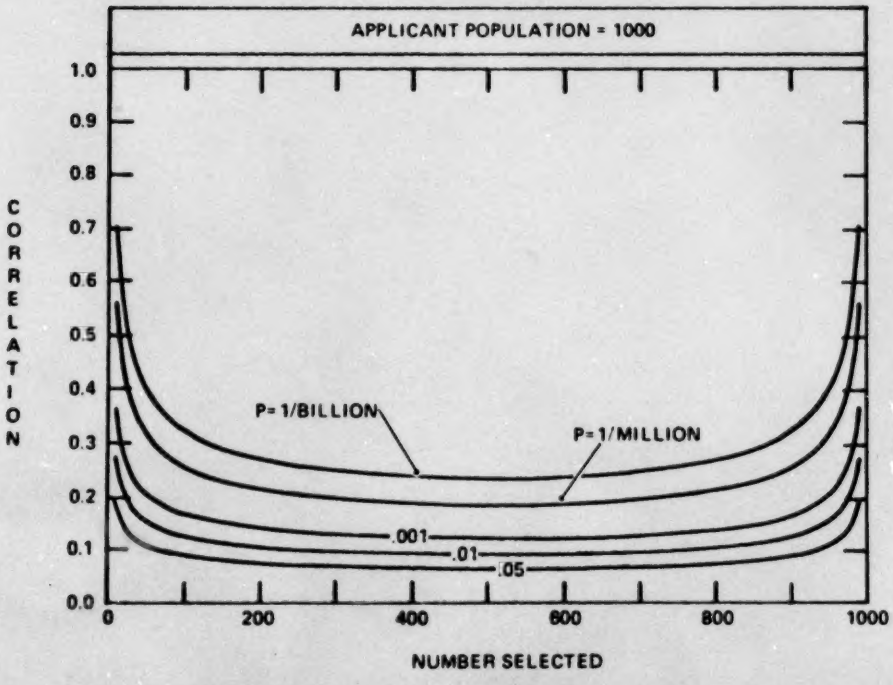
Insert Figure 2 about here

Figure 3 shows why even such small correlations are useful in selection. Superimposed on Figure 1 is the approximate distribution of $\bar{y}_c$ for the case where the applicant pool is 100 and but 10 are to be selected. The mean of this distribution is zero and the standard deviation is $\sqrt{.09} = .3$. The distribution of $\bar{y}_c$ is very tall near its mean when compared

FIGURE 2

PROBABILITY OF $\bar{y}_c > \bar{y}_\rho$ FOR VARYING N,n and ρ

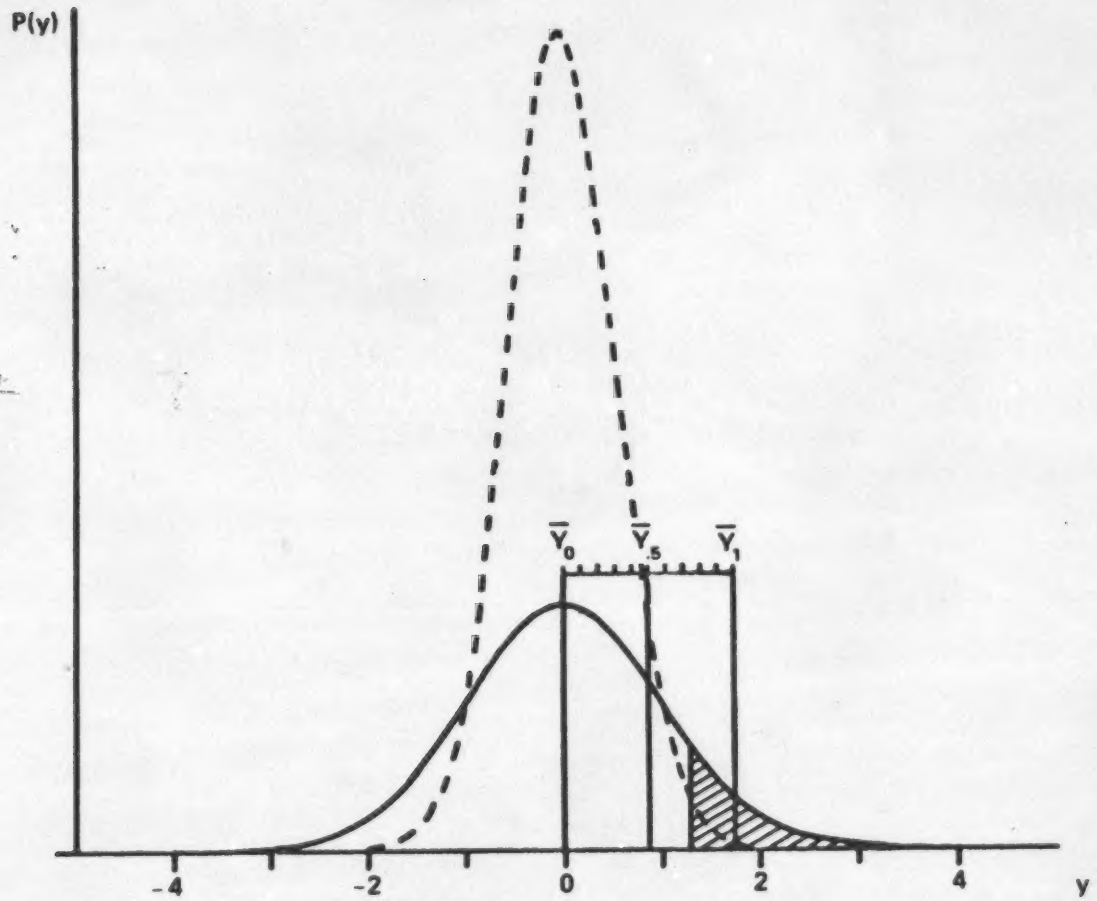




to the distribution of y_i which indicates that most of the $\bar{y}_c$ are close to the average and there are few large deviant values. It is true that the maximum value of $\bar{y}_c$ is $\bar{y}_1$, but the probability of this occurring by chance is approximately $.577 \times 10^{-13}$. It is to this distribution of $\bar{y}_c$ that the value $\bar{y}_\rho$ should be compared and it is clear that even for this very small problem the admissions officer is unlikely to improve on $\bar{y}_\rho$ by chance. For more realistic situations where the applicant pool is much larger, the distribution of $\bar{y}_c$ is even more spiked and thus the probability of exceeding $\bar{y}_\rho$ by chance is even less likely for many reasonable values of ρ.

Insert Figure 3 about here

FIGURE 3
DISTRIBUTION OF MEANS OF RANDOM SELECTIONS
N=100 n=10
DOTTED LINE REPRESENTS DISTRIBUTION OF ALL $\bar{Y}_c$



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